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Final Report for:

**"High Order Accuracy Computational Methods in Aerodynamics
Using Parallel Architectures."**

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Final Report of AFOSR Grant # F49620-95-1-0430

High Order Accuracy Computational Methods in Aerodynamics Using Parallel Architectures

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Abstract

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Statement of Problem Studied

The problems studied in this project involved the development of spectral element methods for simulation of incompressible flows in three-dimensional geometries. This included the development of non-conforming spectral elements which allow for flexible mesh generation and non-propagating refinement (important for adaptivity), the development of fast domain decomposition solvers for large scale parallel applications, and physical applications.

An important component of the work of Tufo was a parallel coarse grid solver which is currently one of the fastest available.

Summary of Research Results

The spectral element method is a high-order weighted residual technique which is essentially a cross between classical spectral and finite element methods. The computational domain of interest is decomposed into relatively large parallelepipeds (bricks) and variables within each subdomain are expressed in terms of high-order tensor-product polynomial bases. The use of tensor-product structures is crucial to the vector performance of the algorithm. Moreover,

the local structure of the high-order brick elements reduces memory latency since pointers are not required for each gridpoint in the domain.

To match complex domain configurations the spectral elements may be slightly deformed, provided there exists an invertible mapping to a cubicle reference domain. While deformation provides a significant increase in the meshing capabilities, it is still somewhat limited when isolated patches of mesh refinement are called for during the course of a simulation. To augment this, a nonconforming spectral element method has been developed which allows for several elements to join to the edge or face of another spectral element. Thus, there can be a rapid transition in spatial resolution without additional mesh deformation.

Since the parallel implementation of the spectral element method is based upon a domain decomposition strategy in which groups of elements are distributed to individual processors, the nonconforming extension to multiple-element interfaces has a non-trivial impact on parallel communication. To simplify the communication software, a general purpose gather-scatter routine was written which could handle arbitrary mesh connectivity through a single, very simple, user interface. This gather-scatter software was originally written in Fortran, but has since been re-written by graduate student H. Tufo in C as a very robust general purpose package which is currently being used by several software development groups.

An additional issue resolved in the thesis of Kruse was the development of a diagonal mass matrix for the nonconforming spectral element methods. This is essential for time-dependent problems as inversion of the mass-matrix is required repeatedly. For his thesis work, Kruse simulated the problem of a high-Reynolds number vortex-pair approaching the ground, which is an external flow problem requiring a highly-varying mesh, and the formation of horse-shoe vortices at the base of an end-mounted cylinder.

The thesis of H. Tufo focussed primarily on fast parallel solution algorithms for elliptic problems as this is a gating issue for incompressible flow simulations in three-dimensions. To improve the iterative performance of the elliptic pressure solve, a domain-decomposition based preconditioner was developed based upon the additive Schwarz procedure of Dryja and Widlund. One computes the solution to the original problem on independent overlapping subdomains (natural extensions of the spectral elements) and adds these solutions together. In addition, a *global* coarse grid problem must be solved which is then interpolated onto the (fine) solution mesh and added to the subdomain solutions. The coarse grid solve is critical to performance of the method; in a standard test case it was found to reduce the overall iteration count by a factor of eight. In parallel, the coarse grid solve is potentially expensive since it requires communication of data amongst all of the processors. We have developed a solver which requires the minimum possible number of message cycles ($\log_2 P$ on P processors) involving messages of length of only $O(n^{\frac{1}{2}})$ for an n -point mesh in two-dimensions and only $O(n^{\frac{2}{3}})$ in three-dimensions. This is a significant reduction over previous algorithms in the literature which typically have $O(n)$ communication complexity. Implementations on the 512 node Intel Paragon at Caltech have proven the viability of this approach for large-scale parallel applications.

List of Publications Supported by this Grant

- G.W. Kruse, *Parallel Nonconforming Spectral Element Solution of the Incompressible Navier-Stokes Equations in Three Dimensions*, Ph.D. Thesis, Division of Applied Mathematics, Brown University, Providence, 1997.
- H.M. Tufo, *Algorithms for Large-Scale Parallel Simulation of Unsteady Incompressible Flows in Three-dimensional Complex Geometries*, Ph.D. Thesis, Division of Applied Mathematics, Brown University, Providence, 1998.
- H.M. Tufo and P.F. Fischer, "Fast Parallel Direct Solvers For Coarse Grid Problems," submitted to *J. Parallel and Dist. Comput.* (1997).

List of Participating Personnel

- G.W. Kruse, Ph.D. student, Research Assistant, Ph.D. degree in 1997.
- H.M. Tufo, Ph.D. student, Research Assistant, Ph.D. degree in 1998.

AUGMENTATION AWARDS FOR SCIENCE & ENGINEERING RESEARCH TRAINING (AASERT)
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- b. F 49620 - 95 - 1 - 0430
Grant Number
- c. _____
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- d. D. Gottlieb
P.I. Name
- e. From: 6/95 To: 6/98
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NOTE: Grant to which AASERT award is attached is referred to hereafter as "Parent Agreement."

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- b. Number FTEGS: 3
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Principal Investigator

Date